Reliability Analysis using Graphical Duration Models

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Abstract

Reliability analysis has become an integral part of system design and operating. This is especially true for systems performing critical tasks such as mass transportation systems. This explains the numerous advances in the field of reliability modelling. More recently, some studies involving the use of Probabilistic Graphical Models (PGMs), a.k.a. Bayesian Networks (BNs), have been proved relevant to represent complex systems and perform reliability studies.

This paper aims to describe a Dynamic PGM (DPGM) designed to model stochastic degradation processes, allowing any kind of state sojourn distributions along with an accurate context description. We meet these objectives using a specific DPGM, namely a Graphical Duration Model (GDM). In this article, we give qualitative and quantitative descriptions of the proposed model and describe how to compute the reliability of the underlying system and some of its classic related metrics.

Finally, we illustrate our approach by applying a GDM in order to perform the survival analysis of railway track supposed to be subjected to one context variable.

1. Introduction

Reliability analysis has become an integral part of system design and operating. This is especially true for systems performing critical applications such as mass transportation systems. Typically, the results of such analysis are given as inputs to a decision support tool in order to optimise the maintenance operations. Unfortunately, in most of cases, the state of the system cannot be exactly predicted. Indeed in most of practical applications, it is uncommon to be able to deterministically model the way a system reaches a failure state. This is one of the reasons which has led to the important development of probabilistic methods in reliability.

A wide range of works about reliability analysis is available in the literature. For instance in numerous applications, the aim is to model a multi-states system and hence to capture how the system state behaves over time or after a given number of sollicitations. This problematic can be partially solved using the Markov framework. The major drawback of this approach comes from the constraint on the state sojourn times which are necessarily exponentially distributed. This issue can be overcome by the use of semi-Markov models [8] which allow to consider any kind of sojourn time distributions. On the other hand, one can be interested in modeling the context impacting on the system degradation . A classic manner to address such an issue consists of using a Cox model or a more general proportional hazard model [5]. Nevertheless, as far as we know, it is unusual to find works considering both approaches at the same time.

More recently, some studies involving the use of Probabilistic Graphical Models (PGMs), a.k.a. Bayesian Networks (BNs), have been proved relevant to represent complex systems and perform reliability studies. For instance, in [2] the authors show how to model a complex system dependability by means of PGMs. In [6] the authors explain how fault trees can be represented by PGMs. Finally, the paper [12] experiments Dynamic PGMs (DPGMs) to study the reliability of a dynamic system represented by a Markov chain. The main interests of a graphical models based approach come from their intuitive use and the numerous associated processing tools (e.g. inference and learning methods).

In this article, we present an original graphical model to capture the behaviour of complex stochastic degradation processes, allowing any kind of state sojourn distributions along with an accurate context description. We achieve to meet these objectives using a specific DPGM called Graphical Duration Model (GDM). This paper is divided into five sections. Section 2 briefly describes the PGMs and DPGMs theory. Then, section 3 introduces GDMs describing the qualitative and quantitative parts of the model. Section 4 details how to achieve reliability studies using GDMs. Then, we illustrate our approach in section 5 by giving an example of railway track survival analysis based on GDMs. Finally, some conclusions and perspectives are discussed in section 6.

2. Probabilistic Graphical Models

2.1. Static definition

Probabilistic Graphical Models (PGMs), also known as Bayesian Networks (BNs) [11], provide a formalism for reasoning about partial belief under conditions of uncertainty. This formalism relies on the probability theory and the graph theory. Indeed, PGMs are defined by a Directed Acyclic Graph (DAG) $\mathcal{G} = (\boldsymbol{X}, \mathcal{E})$ over a sequence of nodes $\boldsymbol{X} = (X_1, \ldots, X_N)$ representing random variables that takes value from given domains $\mathcal{X}_1, \ldots, \mathcal{X}_N$. The set of edges $\mathcal{E}$ encodes the existence of correlations between the linked variables. The strength of these correlations are quantified by conditional probabilities.

A PGM is a pair $(\mathcal{G}, \{P_n\}_{1 \leq n \leq N})$, where $\mathcal{G} = (\boldsymbol{X}, \mathcal{E})$ is a DAG and $\{P_n\}_{1 \leq n \leq N}$ denotes the set of Conditional Probability Distributions (CPDs) associated to each variable X_n and its parents. We refer to the sequence of random variables $\boldsymbol{X}_{\mathrm{pa}_n}$ as the "parents" of X_n in the graph $\mathcal{G}$. Exploiting the conditional independence relationships introduced by the edges of $\mathcal{G}$, the joint probability over $\boldsymbol{X}$ can be economically rewritten with the product form

$$P(x_1, \ldots, x_N) = \prod_{n=1}^{N} P_n(x_n | \boldsymbol{x}_{\mathrm{pa}_n}), \qquad (1)$$

where the general notation $\boldsymbol{x}_S$ (resp. $\boldsymbol{X}_S$) denotes the projection of a sequence $\boldsymbol{x}$ (resp. $\boldsymbol{X}$) over a subset of indices S.

Besides, both the DAG and the CPDs of a PGM can be automatically learnt [10] if some data or experts' opinions are available. Using PGMs is also particularly interesting because of the possibility to propagate knowledge through the network. Indeed, various inference algorithms can be used to compute marginal probability distributions over the system variables. The most classical one relies on the use of a junction tree [7]. In addition, inference in PGMs allows to take into account any variable observations (also called evidence) so as to update the marginal distributions of the other variables.

Finally, note that such modelling are unable to represent dynamic systems (e.g. which contain variables with time dependant distributions). For instance, in reliability analysis, one can be interested in modelling how a system changes from an "up" state to a "down" state over time. For this kind of problems a possible solution consists of using a dynamic extension of PGMs of which an introduction is given in the next section.

2.2. Dynamic probabilistic graphical models

A Dynamic Probabilistic Graphical Model (DPGM) (a.k.a. DBNs, Dynamic Bayesian Network [9]) is a convenient extension of the PGMs formalism to represent discrete sequential systems. Indeed, DPGMs are dedicated to model data which is sequentially generated by some complex mechanisms (e.g. time-series data, bio-sequences, number of mechanical solicitations before the failure of a device,...).

Strictly speaking, a DPGM is a way to extend PGM to model probability distributions over a collection of random variables $(\boldsymbol{X}_t)_{t \in \mathbb{N}^*} = (X_{1,t}, \ldots, X_{N,t})_{t \in \mathbb{N}^*}$ indexed by the discrete-time t. A DPGM is defined as a pair $(\mathcal{M}_1, \mathcal{M}_\rightarrow)$. $\mathcal{M}_1$ is a PGM representing the prior distribution $P_1(\boldsymbol{X}_1) = \prod_{n=1}^{N} P_{n,1}(X_{n,1} | \boldsymbol{X}_{\mathrm{pa}_{n,1}})$. $\mathcal{M}_\rightarrow$ is a particular PGM, called s-slice Temporal Probabilistic Graphical Model (s-TPGM) aiming to define the distribution of $\boldsymbol{X}_t$ given $(\boldsymbol{X}_\tau)_{t-(s+1) \leq \tau \leq t-1}$, where $s \geq 2$ denotes the temporal dependency order of the model. In this paper, we set $s = 2$ such that $\mathcal{M}_\rightarrow$ is a 2-TPGM representing the distribution $P_\rightarrow(\boldsymbol{X}_t | \boldsymbol{X}_{t-1}) = \prod_{n=1}^{N} P_{n,\rightarrow}(X_{n,t} | \boldsymbol{X}_{\mathrm{pa}_{n,t}})$ exploiting the property that given the current process state, the future and past states are independent which is formally denoted by $\boldsymbol{X}_{t-1} \perp\!\!\!\perp \boldsymbol{X}_{t+1} | \boldsymbol{X}_t$

Consequently, it is possible to compute the joint distribution over random variables $(\boldsymbol{X}_t)_{1 \leq t \leq T}$ by simply "unrolling" the 2-TPGM until we have a sequence of length T as follows

$$\begin{aligned} & P((\boldsymbol{X}_t)_{1 \leq t \leq T}) \\ = \; & P_1 \prod_{t=2}^{T} P_\rightarrow(\boldsymbol{X}_t | \boldsymbol{X}_{t-1}) \\ = \; & \prod_{n=1}^{N} P_{n,1}(X_{n,1} | \boldsymbol{X}_{\mathrm{pa}_{n,1}}) \prod_{t=2}^{T} \prod_{n=1}^{N} P_{n,\rightarrow}(X_{n,t} | \boldsymbol{X}_{\mathrm{pa}_{n,t}}). \end{aligned}$$

Finally, as it is possible to consider a DPGM as a big unrolled PGM, these dynamic models inherit some of the convenient properties of static PGMs. On the other hand, performing inference in such models can raise some computation problems if the sequence length is too large. Consequently, specific methods have been developed to partially solve this issue [9]. More precisely, referring to the forward interface $\boldsymbol{I}_t^{\rightarrow}$ as the set of variables which have children in the next slice and not all parents in $\boldsymbol{I}_t^{\rightarrow}$, it can be shown that in a 2-TPGM the future is independent from the past given $\boldsymbol{I}_t^{\rightarrow}$. This property is used as the basis of a generic inference method for DPGMs named interface algorithm. Moreover, coupled with any static inference algorithms (e.g. junction tree), the method is able to compute any sequential probability distribution by updating the forward interface distribution at each step t. In section 4, a simple inference method exploiting the previous remarks is described to compute the reliability of a system represented by a Graphical Duration Model (GDM).

To conclude, as in the rest of the paper we only deal with discrete and finite PGMs, we will use the more explicit notation $P(\boldsymbol{X}|\boldsymbol{Y})$ to denote the CPD of a collection of variables $\boldsymbol{X}$ given a collection of variables $\boldsymbol{Y}$.

3. Graphical Duration Models

Recent works in reliability involving the use of PGMs have been proved relevant since they are particularly suitable to model complex system [2]. Nevertheless in the field of reliability analysis and as far as we know, no author seems to have gone far the use of PGMs to represent more complex models than Markov processes with exogenous constraints [12].

In this article, we propose to extend the variable durations models introduced in [9] to build a comprehensive model able to represent systems having arbitratry survival distributions. We call this particular DPGM a Graphical Duration Model (GDM). In the next paragraphs, we address both the qualitative (i.e. the DAG) and quantitative (i.e. the CPDs) description of a GDM.

3.1. Qualitative definition

The DAG of a GDM is depicted in figure 1 in the form of a 2-TPGM. The solid lines define the basic structure, dashed lines indicate optional items and bold edges characterise dependencies between time slices. The model consists in the following two main parts :

1. The collection $(X_t)_{1\leq t\leq T}$ represents the system state over a sequence of length T.

2. The collection $(X_t^D)_{1\leq t\leq T}$ represents the remaining time before a system state modification. More clearly, we refer to the random variable X_t^D as the remaining sojourn time in the current system state. These variables are called duration variables.

Optionally, it is possible to introduce a context description of the studied system by means of a prior PGM $\mathcal{M}_{\boldsymbol{Z}_t}$. It aims to define the distribution of a collection of context variables (covariates) $\boldsymbol{Z}_t = (Z_{p,t})_{1\leq p\leq P}$. In figure 1, the optional outgoing edge between $\mathcal{M}_{\boldsymbol{Z}_t}$ and X_t (resp. X_t^D) signifies that there is at least one variable $Z_{p,t}$ which has an outgoing edge to X_t (resp. X_t^D). Moreover, it is possible to complete the qualitative definition of GDMs by adding in each time slice a transition variable J_t. It is a binary variable introduced to explicitly characterise when the system state changes. Indeed when the previous transition indicator is enabled (i.e. $J_{t-1} = 1$), the system state will change at time t. On the other hand, while $J_t = 0$, the system remains in the same state. This variable is clearly optional since it does not alter the independence relationships but appears to be convenient for further generalization and CPDs definition

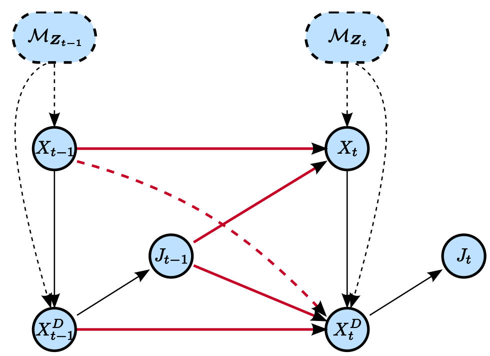

Figure 1. 2-TPGM associated to a GDM.

Besides, the DAG of a GDM shows that the current system state X_t depends on the previous system state X_{t-1}, the previous remaining duration X_{t-1}^D and the current context $\boldsymbol{Z}_t$. On the other hand, the current duration variable X_t^D is dependant on the previous duration variable X_{t-1}^D, the context $\boldsymbol{Z}_t$, the current state X_t and optionally on the previous state X_{t-1}. Consequently, the process (X_t) (resp. (X_t^D)) is not Markovian since $X_{t-1} \not\perp\!\!\!\perp X_{t+1}|X_t$ (resp. $X_{t-1}^D \not\perp\!\!\!\perp X_{t+1}^D|X_t^D$). On the other hand, the GDM structure leads to

$$(\boldsymbol{Z}_{t-1}, X_{t-1}, X_{t-1}^D, \boldsymbol{Z}_t) \perp\!\!\!\perp (\boldsymbol{Z}_{t+1}, X_{t+1}, X_{t+1}^D)|(X_t, X_t^D). \tag{2}$$

Thus, the set $\{X_t, X_t^D\}$ is the forward interface for a GDM. Consequently, the process (X_t, X_t^D) engendered by a GDM is Markovian and generalises the recently studied discrete semi-Markovian processes [1]. Indeed, this approach allows to specify arbitrary state sojourn time distributions by contrast with a classic Markovian framework in which all durations have to be exponentially distributed. This modelling is then particularly interesting as soon as the question is to capture the behaviour of a given system subjected to a particular context and a complex degradation distribution.

As the qualitative part of GDMs has now been presented, let address the review of the quantitative part.

3.2. Quantitative part

The following paragraphs address the specification of each CPD involved in a GDM except those about the distribution of $\boldsymbol{Z}_t$ since the system context modelling is strongly application dependant. Hence in the sequel, we suppose that the PGM governing the distribution of each $\boldsymbol{Z}_t$ is known.

3.2.1 System state CPDs

As in this paper only discrete and finite systems are considered, we note $\mathcal{S} = \{1, \ldots, K\}$ the set of the K different system states.

Firstly, The CPD associated with the distribution of the initial system state given the context is defined as follows :

$$P(\underbrace{X_1 = i}_{\text{initial state}} \,|\, \underbrace{\boldsymbol{Z}_1 = \boldsymbol{z}}_{\text{context}}) = p_{X_1}(i|\boldsymbol{z}),$$

where $p_{X_1}(\boldsymbol{z}, \cdot)$ is the discrete and finite probability distribution of initial system state in the context $\boldsymbol{z}$.

Then, it is necessary to define the transition CPD from one state to another. As a transition occurs at time t if and only if the variable $J_{t-1} = 1$, we have

$$P(\underbrace{X_t = j}_{\text{current state}} \,|\, \underbrace{X_{t-1} = i}_{\text{previous state}}, \underbrace{J_{t-1} = 1}_{\text{transition at time } t}, \boldsymbol{Z}_t = \boldsymbol{z}) = A(i, \boldsymbol{z}, j),$$

where $A(\cdot, \boldsymbol{z}, \cdot)$ is a stochastic $K \times K$ matrix dependant on $\boldsymbol{z}$. Basically, $A(i, \boldsymbol{z}, j)$ is merely the probability that the system goes to state i from state j given the context $\boldsymbol{z}$. We refer to $A(\cdot, \boldsymbol{z}, \cdot)$ as the static transition matrix of the system since it is time-independent.

On the other hand, while there is no transition, i.e. $J_{t-1} = 0$, the system deterministically remains in the previous state i. Therefore, the corresponding static transition matrix is reduced to identity :

$$P(X_t = j | X_{t-1} = i, \underbrace{J_{t-1} = 0}_{\text{no transition}}, \boldsymbol{Z}_t = \boldsymbol{z}) = \begin{cases} 1 & \text{if } j = i, \\ 0 & \text{otherwise.} \end{cases},$$

which makes the CPD deterministic given $J_{t-1} = 0$ whatever the context z.

3.2.2 Duration CPDs

The initial duration CPD describes the sojourn time distributions for each initial state given the context $\boldsymbol{z}$, such that

$$P(\underbrace{X_1^D = d}_{\text{sojourn time in state } i} | \boldsymbol{Z}_1 = \boldsymbol{z}, X_1 = i) = p_{X_1^D}(d|\boldsymbol{z}, i), \quad (3)$$

where $p_{X_1^D}(d|\boldsymbol{z}, i)$ is the probability to remain d time units in the state i given the context $\boldsymbol{z}$. Besides, as in this paper we are only dealing with discrete and finite PGMs, $p_{X_1^D}(\cdot|\boldsymbol{z}, i)$ has also to be a finite and discrete probability distribution function which is not natural for time distributions. Basically, we overcome this issue by setting an upper time bound $D_{\max}$ large enough compared to the dynamic of the studied system. For instance, suppose we would like to carry out a reliability analysis on a particular photocopier model in nominal use. Setting $D_{\max} = 100$ years seems to be more than enough regarding the average lifetime of such devices. It follows that $p_{X_1^D}(\cdot|\boldsymbol{z}, i)$ becomes a finite distribution taking on $D_{\max}$ values for each state i and each context $\boldsymbol{z}$.

The transition duration CPD plays an analogous role except it has also to update the remaining time to spend in the current state at each sequence step. Indeed, while the remaining previous duration is greater than one (i.e. $X_{t-1}^D > 1$), the remaining sojourn time is deterministically counted down, hence

$$P(\underbrace{X_t^D = d}_{\substack{\text{current} \\ \text{remaining time}}} | X_{t-1} = j, \underbrace{X_{t-1}^D = d'}_{\substack{\text{previous} \\ \text{remaining time}}}, \underbrace{J_{t-1} = 0}_{\text{no transition}}, \boldsymbol{Z}_t = \boldsymbol{z}, X_t = i)$$
$$= \begin{cases} 1 & \text{if } d = d' - 1 \text{ and } d' \geq 2 \text{ and } i = j, \\ 0 & \text{otherwise.} \end{cases}.$$

On the other hand, when the previous remaining duration reaches the value one, a transition is triggered to occur at time t and a duration for the new current state is drawn according to the following CPD :

$$P(\underbrace{X_t^D = d}_{\substack{\text{new} \\ \text{sojourn time}}} \,|\, \underbrace{X_{t-1} = i}_{\text{previous state}}, \underbrace{X_{t-1}^D = 1, J_{t-1} = 1}_{\text{transition triggered}}, \boldsymbol{Z}_t = \boldsymbol{z}, \underbrace{X_t = j}_{\substack{\text{new} \\ \text{current state}}})$$
$$= p_{X_t^D}(d|i, \boldsymbol{z}, j).$$

where $p_{X_t^D}(d|i, \boldsymbol{z}, j)$ is the probability to remain d time units in the state j while the previous state was i given the context $\boldsymbol{z}$. The remarks made concerning the initial duration distribution function $p_{X_1^D}$ stay valid for $p_{X_t^D}$ especially since the dependency on the previous state is optional. In this latter case, $p_{X_1^D}$ and $p_{X_t^D}$ are identical.

Finally, The discrete-time assumption laid on by the DPGM formalism can be easily overcome. Indeed, authors in [3] present a survey of discrete lifetime distributions and explain how to derive usual continuous ones (e.g. exponential, Weibull) in the discrete case.

3.2.3 Transition CPD

J_t is the random variable characterising transitions between two different system states. More precisely, when $J_t = 1$, a transition is triggered at time t and the system state changes at time $t+1$. The system state remains unchanged while $J_t = 0$. Besides, a transition is triggered at time t if and only if the current remaining duration reaches the value one. Consequently, the CPD of J_t is deterministic and defined merely by

$$P(J_t = 1 | X_t^D = d) = \begin{cases} 1 & \text{if } d = 1, \\ 0 & \text{otherwise.} \end{cases}$$

4. Reliability analysis using GDMs

Let assume that the set of system states $\mathcal{S}$ is partitioned into two sets $\mathcal{U}$ and $\mathcal{D}$ (i.e. $\mathcal{X} = \mathcal{U} \cup \mathcal{D}$ with $\mathcal{U} \cap \mathcal{D} = \varnothing$), respectively for "up" states and for "down" states (i.e. OK and failure situations). The discrete-time system reliability is then define as the function $R : \mathbb{N}^* \mapsto [0,1]$ where $R(t)$ represents the probability that the system has always stayed in an up state until moment t, i.e. $R(t) = P(X_1 \in \mathcal{U}, \ldots, X_t \in \mathcal{U})$. In addition, it is possible to derive some interesting metrics such as the failure rate or the MTTF (cf. [3] for details) from the reliability definition. As the failure rate and the MTTF can be expressed using the reliability, the objective is to build an algorithm able to compute $R(t)$. Hence, this issue boils down to an inference problem, i.e. computing $P(X_1 \in \mathcal{U}, \ldots, X_t \in \mathcal{U})$. To that end, the following result can be shown :

Proposition 1 *Let $(X_\tau)_{1 \leq \tau \leq t}$ be the collection of system state variables represented by a GDM. Then,*

$$P(X_1, \ldots, X_t) = \alpha_1 \cdot \prod_{\tau=2}^{t} \Lambda_\tau, \ \forall t \geq 2, \tag{4}$$

where $\alpha_1 = P(X_1)$ and $\Lambda_t = P(X_t | X_{t-1})$. $(\Lambda_t)_{t \geq 2}$ is the collection of system transition matrices at each time t.

Basically, α_1 is the initial system state distribution and Λ_t is the matrix describing the system transition distribution from one state to another at step t. Hence this result shows that the system state process generated by a GDM can be represented as a Markov chain with a time dependant transition matrix. Besides, using basic probability manipulation rules, it is straightforward to see that

$$\alpha_1 = \sum_{\boldsymbol{Z}_1} P(\boldsymbol{Z}_1) \cdot P(X_1 | \boldsymbol{Z}_1), \tag{5}$$

$$\Lambda_t = \sum_{X_{t-1}^D, J_{t-1}, \boldsymbol{Z}_t} P(X_{t-1}, X_{t-1}^D) \cdot P(J_{t-1} | X_{t-1}^D) \cdot P(\boldsymbol{Z}_t) \cdot P(X_t | X_{t-1}, J_{t-1}, \boldsymbol{Z}_t) \tag{6}$$

We exploit the proposition result in an interface algorithm coupled with any given static inference methods able to answer any queries of the form $P(\boldsymbol{X} | \boldsymbol{Y})$. The corresponding algorithm is described by

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Compute the initial interface distribution P(X_1, X_1^D)
Find out α_1 = P(X_1) according to equation (5)
for t = 2 to T do
    Compute Λ_t = P(X_t | X_{t-1}) according to (6)
    Update the interface distribution P(X_t, X_t^D)
    Find out R(t) = α_1(U) ∏_{τ=2}^{t} Λ_τ(U, U).
end for
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In practice, we use a variable elimination algorithm [4] due to the fact that it is easy to understand, to implement and does not involve useless complex preprocessing to meet our objective.

In the next section, we illustrate our approach explaining how to use GDMs to model the behaviour of a concrete system and compute its reliability and its related metrics.

5. Application

We present some results from a railway track lifetime study performed by means of GDMs modelling. These results are very useful to optimise the maintenance operation which is especially important for systems performing critical applications such as in the mass transportation domains.

Indeed, the continuous increasing of traffic can lead to rail break growths. For safety reason, restrictive exploitation rules are defined (e.g. a train can run on a breaking rail but only after an enforcement process and at reduced speed) increasing delays and hence diminishing the service quality. Moreover, expensive corrective maintenance costs are needed to make up for this kind of failure. For these reasons, efforts are being made for the application of reliability-based maintenance optimisation of railway infrastructures. The underlying idea is to reduce the operation and maintenance expenditures while still assuring high safety standards.

Note that in this paper, we focus only on the reliability analysis of the railway track and let maintenance modelling for further works.

5.1. Variable definitions

We now define the meaning of the different variables involved in the GDM modelling our railway case study.

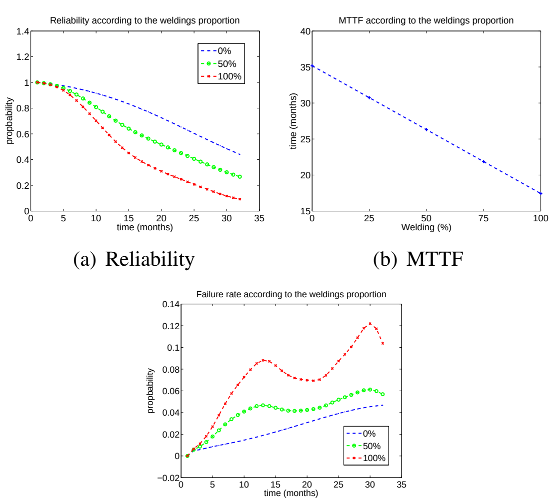
$Z_{1,t}$ represents the type of the material installed in the studied track section : "homogeneous rail" (R), "welding" (W) and then its associated definition domain is $\mathcal{Z}_1 = \{\mathrm{R}, \mathrm{W}\}$. Basically, when a serious damage is detected, maintenance operators proceed to a local rail renewal. To that end, they perform an aluminothermic welding which leads to two welding joints on the rail section tips. As welding lifetime is lower than the rail lifetime, most of new damages occur around the formers. As a consequence, the more welding are installed, the weaker the global reliability of the studied track is.

X_t is the system state, namely the state of the considered railway track. We assume three states of degradation: "no defect" (N), "minor defect" (D), "critical failure" (F) and then $\mathcal{S} = \{\mathrm{N}, \mathrm{D}, \mathrm{F}\}$. Suppose in addition that the first two states do not bring on service disturbances or safety problems like the last one. That is why we set $\mathcal{U} = \{\mathrm{N}, \mathrm{D}\}$ and $\mathcal{D} = \{\mathrm{F}\}$.

X_t^D is the duration variable. In this case, we use a GDM with no edge between X_{t-1} and X_t^D, so that the duration distribution is only dependent on current state X_t.

5.2. Results

The GDM used in this application has been implemented in a MATLAB® environment, completed by the free Bayes Net Toolbox (BNT) . The transition matrices along with all the sojourn time distributions have been learnt from railway track maintenance feedback experience databases. The estimation method previously presented has been used to compute the reliability and the related metrics for the considered system. The results are depicted in figures 2. These figures characterise the degradation process of the studied railway track according to the considered proportion of weldings and hence evaluating the impact of the context. Thereby, such information are essential to set up relevant maintenance policies. Finally, we can observe the linear decrease of the MTTF according to the welding proportion. However, the proof of this experimental result is deferred to a further paper.



(a) Reliability (b) MTTF

(c) Failure rate

Figure 2. Railway track reliability.

6. Conclusion

The proposed method based on the GDMs aims to study the behaviour of a complex system. Our approach turns to be a satisfying and a comprehensive solution to model and estimate the reliability of a complex system. Indeed, the proposed modelling is generic since it is possible to take into account the context of the system along with an accurate description of its survival distributions. In addition as this work is based on graphical models, the underlying approach is intuitive and easily generalisable. The encouraging results presented in this paper confirm that GDMs are competitive reliability analysis tools for practical problems. Finally in future works, we will address the problem of maintenance modelling based on system represented by GDMs.

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