Monitoring rail corrugation in the mileage-wavelength domain using the ARCAP method

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ABSTRACT
Rail corrugation is an oscillatory wear of rail surface due to the interaction between rail and wheel. Standard signal processing approaches to corrugation monitoring, as devised in the European standards and in use in railway networks, are designed in the mileage or wavelength domain. Time-frequency analysis computes the energy distribution of the signal in the joint time and frequency domains. Such a representation highlights the time-evolving characteristics of the analysed signal. Time-frequency methods are thus particularly suited to the analysis of non-stationary signals.
We propose here to perform a time-frequency based diagnosis of rail corrugation. Mileage is processed in lieu of time and spatial wavelength is processed in lieu of frequency. This approach provides an easy-to-read map of the corrugation modes evolution with mileage.
We put a particular emphasis on the so-called ARCAP method. This method assumes a model of the corrugation data as a sum of sinusoids. Such a model is well fitted to the data. This assumption allows thus for a highly efficient modal analysis of the corrugation pattern. Both the depth and frequency of each mode are accurately estimated along mileage.
The ARCAP method outperforms the standard mileage or wavelength domain methods in localizing and characterizing corrugation.

INTRODUCTION
Rail corrugation is an oscillatory wear phenomenon occurring mainly on urban railway transportation networks. It is a consequence of the mechanical interaction between the passing train and the rail depending mainly on the oscillating characteristics of the train and on the mechanical properties of the rail and wheels. Rail corrugation can be considered as a closed loop combination of a wavelength fixing mechanism and a damage mechanism (Grassie, 2005). The consequence is an oscillatory vertical wear of the rail along the longitudinal axis, as it can be seen on Figure 1.
Rail corrugation is a source of noise and vibration for the surrounding neighbourhood and passengers. It reduces the lifetime of both wheels and rails but also of other components of tracks and bogie. For environmental as well as economical issues, reducing - if not annihilating - rail corrugation is thus a matter of great concern for railway networks.

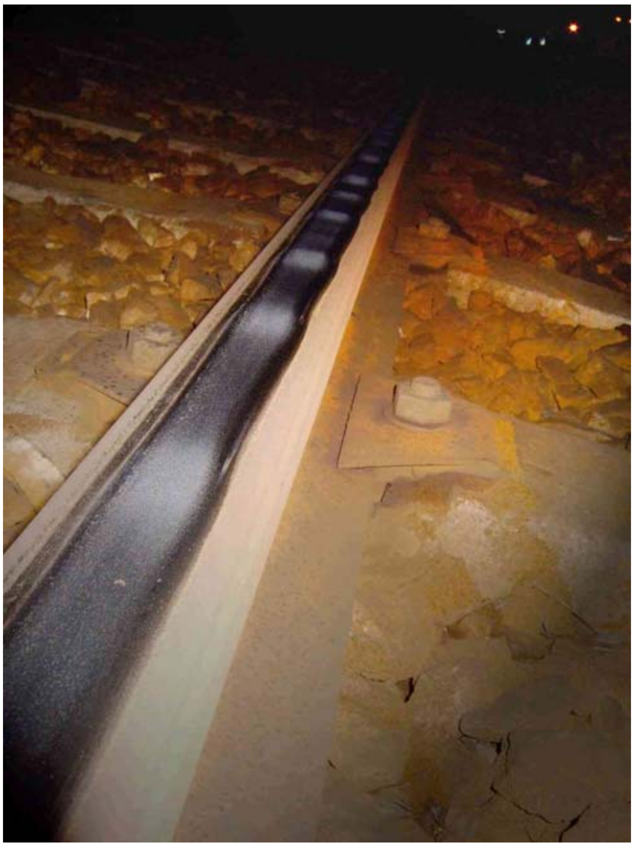

Figure 1: Corrugation on the line A of the Paris RER network between Fontenay-aux-Roses and Robinson stations.

Many studies have been dedicated to the comprehension of the rail corrugation phenomenon (see for example Sato et al, 2002 or Grassie, 2005). Even though chemistry, matter physics and mechanics can help devising new railtrack materials with a longer lifetime, it is almost impossible to avoid rail corrugation. Diagnosis have thus to be performed on existing railway network so that to detect and cure rail corrugation following a determined maintenance strategy. In this paper the authors adopt a signal processing point-of-view on the diagnosis of rail corrugation.
From the analysis of the measurement signal of a segment of railway track, the diagnosis of corrugation consists in

1. detecting corrugation pattern;
2. characterizing corrugation.

European standard prEN 13231-3 is currently being written to provide the rail industry with recommendations on corrugation diagnosis. Under this standard both steps have to be performed in the original domain of measurement, namely the *mileage domain*. It is advised to perform corrugation detection by applying a threshold either to an estimation of the peak-to-peak (PTP) amplitude or to the root-mean-square (RMS) magnitude. Operators of the Parisian rail network RATP analyse also the spectrum of the measurement signal, a distribution of the signal energy in the wavelength domain. Spectral analysis of rail roughness is actually advised by European standard prEN 15610, devoted to noise monitoring. Since corrugation is also a source of train vibrations and noise, spectral analysis of corrugation signal happens to be of interest for this matter.
Methods based on a single dimension representation of the signal (mileage or wavelength) miss the complementary information which can be of crucial interest. Suppose the magnitude of the measurement signal is wavelength-dependent or, inversely, the wavelength content of the signal evolves with the mileage. A mileage-only representation of the signal fails to provide information about the wavelength content whereas a wavelength representation (spectrum) is enable to display the mileage-varying information.

Time-frequency[1] tools (Flandrin, 1999) are designed to overcome this fundamental issue. Caprioli et al (2007) have conducted a comparative study of the Short-Time Fourier Transform and the wavelet packets for rail inspection and track maintenance. This is, up to our knowledge, one of the few examples where authors tackle the rail corrugation diagnosis problem within a time-frequency framework. Since the methods in study are based on Fourier analysis and its generalization to wavelet analysis they suffer from the inherent trade-off between mileage and wavelength resolution: in order to keep the mileage-varying wavelength information the analysis of the signal is performed on a short-length sliding window which induces a deteriorated wavelength accuracy. This drawback can make such methods dramatically inefficient if several close wavelengths are superimposed in the corrugation signal.
Parametric methods such as Auto Regressive (AR) modelling (Marple, 1987) are known to provide a better frequency resolution than classical Fourier methods since additional a priori information about the signal is taken into account. However the AR spectral estimator is biased. In order to overcome this drawback Padovese et al (1996) proposed to combine the wavelength estimation provided by the AR method with a Capon filtering (Capon, 1969) of the signal. The Capon filter provides a highly accurate estimation of spectral peaks. In this paper a description of the so-called ARCAP (AR-Capon) method and its application to the problem of corrugation diagnosis is proposed.

CURRENT APPROACH TO THE DIAGNOSIS OF RAIL CORRUGATION

In this section the procedure for diagnosis of corrugation as currently performed at RATP is described.

Data Acquisition

The measurement device currently in use for inspecting railtrack and performing corrugation diagnosis on the RATP network was proposed by Levy (1989). A picture and a schematic representation of the device are shown on Figure 2.

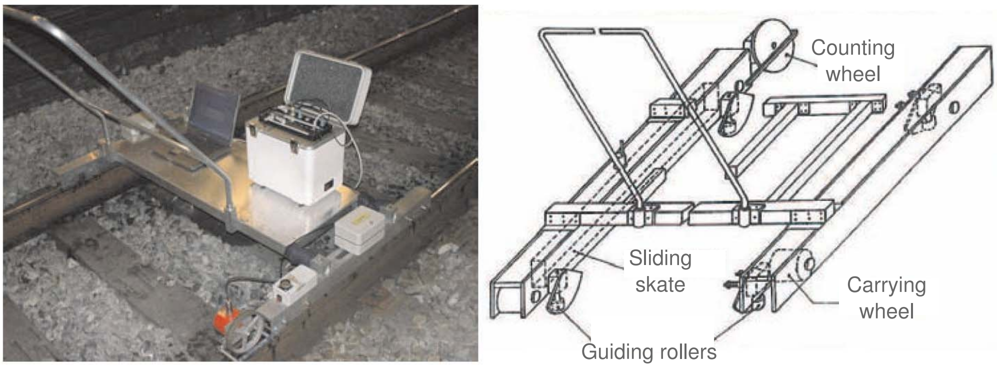

Figure 2: Measurement device. Courtesy of RATP.

An Eddy current sensor is fixed on a 80cm long sliding skate. The data acquisition system is carried on a trolley. The trolley is moved at human walking speed so that the skate slides along the inspected rail. The mileage of the rail to the middle of the pad is measured every 2mm and recorded on the embedded laptop.

Data processing

Pre-filtering

The measurement signal is filtered with a high-pass zero-phase non-causal Butterworth filter of order 4 in order to remove the dynamic of the measurement device. The cut-off wavelength is of 1m.
Figure 3 shows a corrugation signal collected using the device of Figure 2. Let focus on the bottom figure which shows the pre-filtered signal. There is a highly energetic waveform (approximately 2mm deep) around 8m which has no physical meaning in regards of the shape of the rail. It might be due to an abrupt move of the trolley. After this peak and until 100m the depth is lower than 0.05mm. There is no damaging

[1] by convention we will refer in this article to the term *time-frequency analysis* even though *mileage-wavelength analysis* would be more appropriate

corrugation on this section of rail. Corrugation occurs between 100m and 300m. In this section the depth of the wear pattern increases up to 0.4mm.

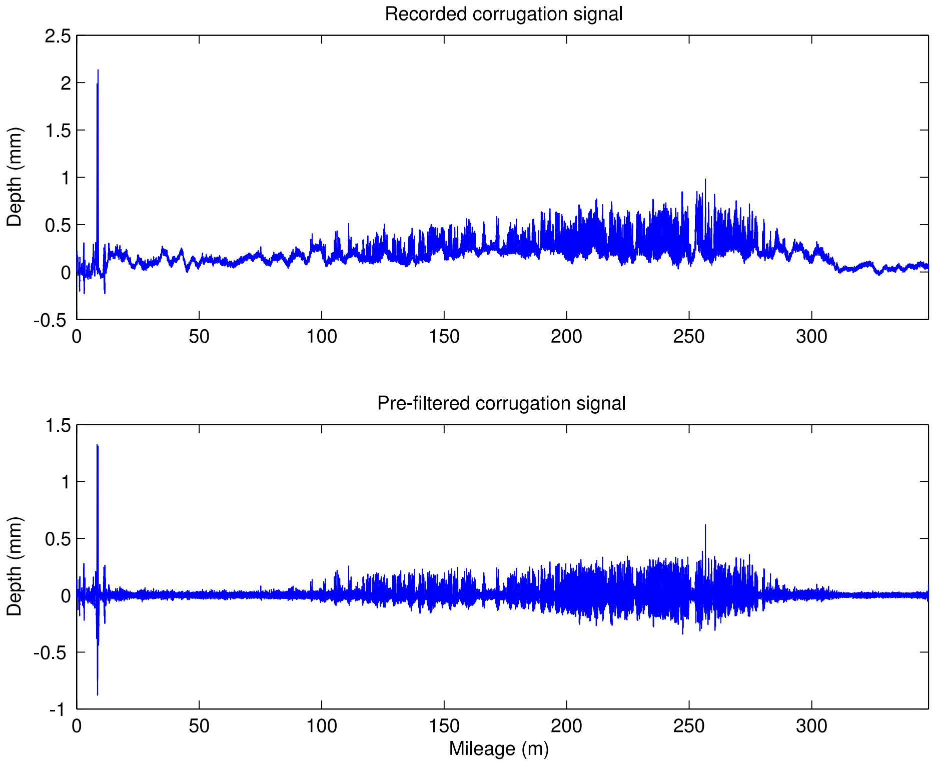

Figure 3: Corrugation signal. Raw signal (top) and pre-filtered signal (bottom). Data recorded on the 15th of May 2007 at pK 15800, inner rail of track 2bis, line B of the Parisian RER network. The segment is 347.646 meter long, decreasing pK. The sampling frequency is 2mm.

Although a visual inspection of the pre-filtered signal allows to guess where corrugation occurs, it is impossible to characterize the corrugation modes and the contribution of each mode to the total wear. The European standard recommends to perform a band-pass analysis of the pre-filtered signal so that to isolate the various corrugation modes with specific mechanical sources.

Band-pass analysis

According to the European standard, the pre-filtered signal analysis must be performed in the following four wavelength bands:

1. 10-30mm;
2. 30-100mm;
3. 100-300mm;
4. 300-1000mm.

Note that operators at RATP do not perform rail inspection in the 10-30mm band since corrugation is not observed on the RATP network in this band.

The signal is filtered in each band using zero-phase Butterworth band-pass filters of order 4. These wavelengths have been chosen in order to isolate corrugation patterns of different sources. Detection is performed in each wavelength band by applying a threshold either to the RMS or to the PTP amplitude of the signal.

Table 1 shows the result of the acceptance control performed on the signal of Figure 3. For each criterion and for each wavelength band the percentage of rail section off-tolerance is computed according to the threshold levels set by the European standard. In each analysis band and according to the chosen criterion (RMS or PTP), Table 1 indicates the percentage of rail length that should be grinded. It appears that whatever the chosen criterion corrugation occurred on this section mainly in the 30-100mm band.

Table 1: Acceptance analysis of signal of Figure 3. Root mean Square (RMS) and Peak-to-peak amplitude (PTP) analysis in the three wavelength bands inspected at RATP.

Wavelength band	RMS threshold	RMS off-tolerance	PTP threshold	PTP off-tolerance
30-100mm	0.004	87.2 %	0.01	71.1 %
100-300mm	0.012	55.5 %	0.03	48.1 %
300-1000mm	0.04	13.1 %	0.1	18.4 %

It is also recommended by the European Standard on noise control to estimate the RMS level from the observation of the periodogram estimate of the power spectral density of the pre-filtered signal. In order to set up a threshold on the basis of perceptual acoustic level tolerance, the third octave spectrum must be observed.

Spectral resolution

Following the European standard, the accuracy of the corrugation analysis in terms of wavelength estimation is limited to the bandwidths. A refined band-pass analysis of the signal should provide additional information about the oscillation modes.

Let illustrate our purpose with the signal segment of Figure 4. It is a 8.19 meter long section of the signal of Figure 3 located 152m away from the initial measurement point.

This segment exhibits a main oscillatory pattern with a periodicity of about 600mm and a superimposed faster oscillatory pattern with a periodicity of about 100mm. The 600mm oscillatory pattern corresponds to the distance between sleepers. The corrugation mode is the 100mm component. Note also that the depth collapses between 155.5m and 157m.

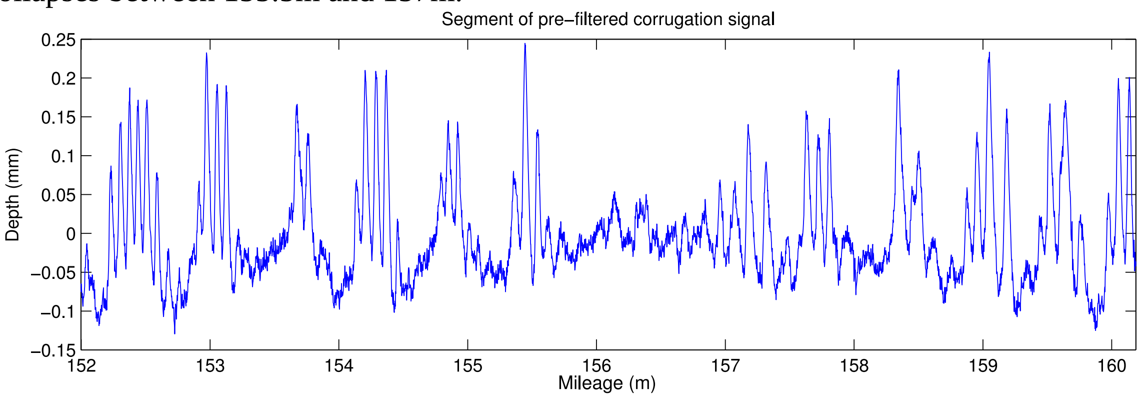

Figure 4: A 8.19 meter long segment of signal of Figure 3.

Figure 5 shows the third-octave spectrum of this segment. The acceptance limit defined from psycho-acoustics considerations is superimposed to the spectrum.

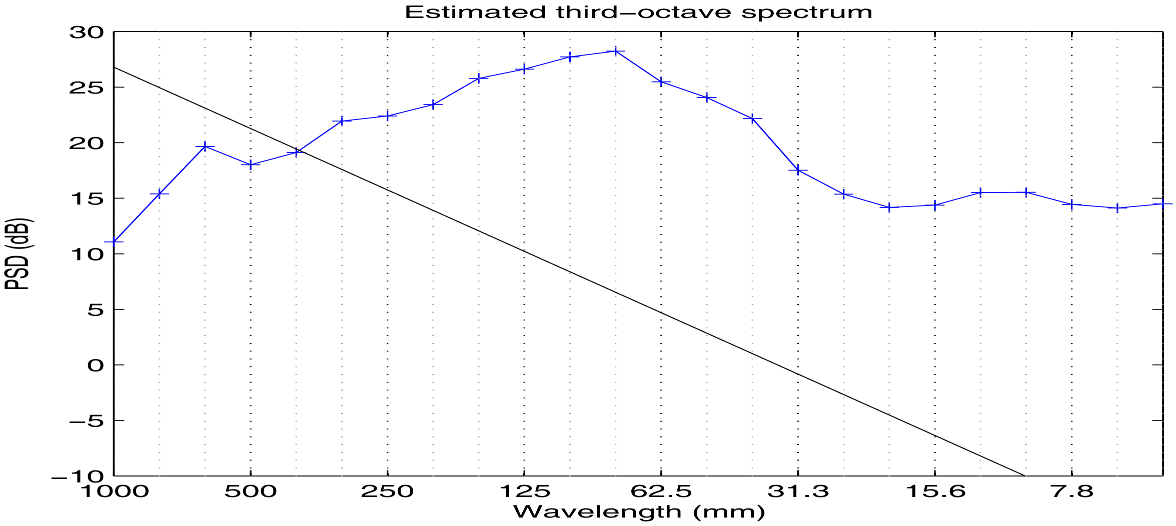

Figure 5: Third-octave spectrum estimate of segment of Figure 4. The spectrum estimator (crossed blue line) is the Welch periodogram computed with a 500 sample long Hanning window (1m) with a 499 sample long overlapping on a FFT grid of 1024 points. Black line is the psycho-acoustics acceptance limit.

The third-octave spectrum exhibits a main peak around 100mm and a second peak around 600mm as observed from the mileage-only representation of the signal (Figure 4). It can be seen that the spectrum go beyond the acceptance level for wavelength lower than 400mm. According to the acceptance procedure of European standard, it can be concluded that if this section of rail had been refurbished, it should be grinded again because corrugation modes in the wavelength bands 100-300mm and 30-100mm would have not been properly cured.

The analysis of the third-octave spectrum has lead to conclude that corrugation occurred in the 30-100mm and 100-300mm wavelength bands. Let move a step beyond and estimate more accurately what is the corrugation wavelength.

Figure 6 shows the spectrum of Figure 5 on a linear Fast Fourier Transform (FFT) grid. The wavelength axis has been artificial stressed in order to give the same visual emphasis to each of the four wavelength bands of interest. Because of the integration in the third-octave bands, the third-octave representation of the spectrum underestimates the 600mm component relative magnitude.

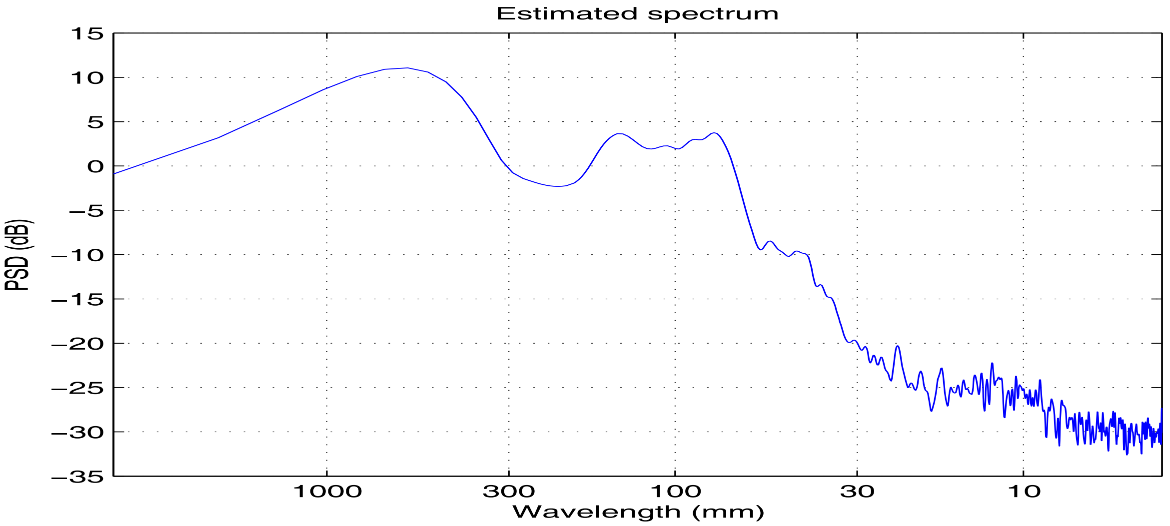

Figure 6: Linear FFT grid spectrum of segment of Figure 4. The x-axis has been artificially stressed in order to exhibit the wavelength bands of interest.

Note that the 100mm peak is actually split in two peaks which can not be observed on the third-octave spectrum. This may indicate that two corrugation modes have actually appeared on this rail segment in both 30-100mm and 100-300mm bands.

<u>Combining wavelength and mileage information</u>

By definition, the spectrum displays the energy content of the signal along the whole rail section length. This means that whatever the chosen visualization of the signal spectrum (Figure 5 or 6), there is no available information about the evolution of the corrugation mode with mileage. The wavelength evolution with mileage is only available up to the section length, which is in this case of 8.19m.
A remedy to this drawback is to compute the spectra of contiguous short-term segments of the analysed signal. The resulting bi-dimensional representation of the signal is the Spectrogram.
The spectrogram of the signal of Figure 4 is shown on Figure 7. The power spectral density (in dB) is encoded by the colour scale of the representation displayed on the right-hand side of the figure. Such a representation provides information about the mileage evolution of the wavelength content. Note for example the dramatic decrease of the energy around 155.5m which is observable on the mileage representation of the signal on Figure 4.
The mileage-evolution of the 100mm and 600mm components can also be observed from the spectrogram. It is of interest to notice that what we refer to as a 100mm component from the observation of the spectra is actually evolving along mileage from 80mm around 152m to 200mm around 159m. This explains why the more accurate spectrum of Figure 6 exhibits two peaks. We can see here again how the bi-dimensional analysis of the corrugation signal can avoid a wrong interpretation of the wavelength only representation.

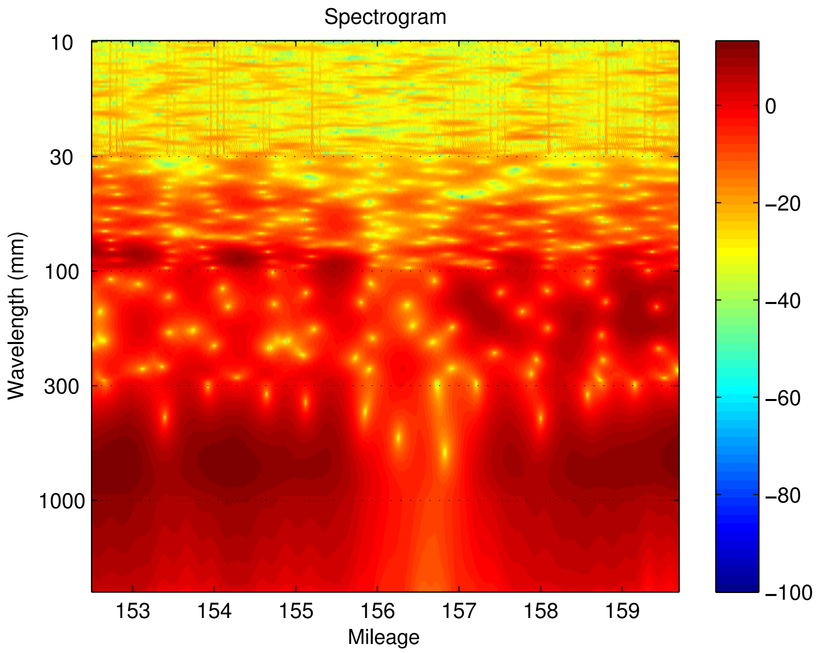

Figure 7: Spectrogram of the signal segment of Figure 4, computed with a 1 meter-long Hanning window and a 0.995 meter overlapping. The signal has been down-sampled by a factor of 2.5 (sampling period: 5mm).

DIAGNOSIS OF RAIL CORRUGATION IN THE MILEAGE-WAVELENGTH DOMAIN

The spectrogram is the seminal method of time-frequency analysis. Many more elaborate methods of time-frequency analysis have been proposed in the literature (Flandrin, 1999). Concerning diagnosis of corrugation, Caprioli et al.(2007) have proposed a comparative study of two famous time-frequency tools: the Short-Time Fourier Transform and the wavelet packets. This is, up to our knowledge, one of the few examples where authors deliberately choose to tackle the rail corrugation diagnosis problem within a time-frequency framework. Since the methods in study are based on Fourier analysis and its generalization to wavelet analysis they suffer from the inherent trade-off between time and frequency resolution: in order to keep the mileage-varying frequency information the analysis of the signal is performed on a short-length sliding window which induces a low frequency resolution. This drawback can make such methods dramatically inefficient if several close frequencies are combined in the corrugation signal.

We propose here to apply a model-based time-frequency tool called ARCAP and introduced by Padovese et al. (1996). The model-based approach allows for a higher accuracy of the estimation by introducing additional information about the spectral content of the signal. If the analysed data fits the model the above mentioned time-frequency uncertainty drawback can be partially overcome. In the ARCAP method the corrugation wavelengths are estimated using an Auto Regressive (AR) method and the spectral magnitudes are computed at those estimated wavelengths using the Capon (CAP) filter.

The AR method

AR approaches of spectral estimation (Marple, 1987) are based on the modelling of the analysed signal as the output of an all-pole system [16] which admits a white noise as input. The transfer function of an all-pole system is a polynomial rational function with a constant numerator. The poles locate the resonance wavelengths of the system. They are by definition the zeroes of the denominator polynomial. The spectrum of the output of the system (the analysed signal) is a function of the AR polynomial coefficients. An AR spectral estimator is thus a parametric estimator in the sense that the spectral estimation is reduced to the estimation of a few parameters (the AR coefficients). The number of parameters (which is also the number of poles) is the order of the model.

The mechanical interaction that generates corrugation can be modelled by a resonant system (Hempelmann & Knothe, 1996). More specifically, the oscillatory wear pattern can be modelled by a sum of sinusoids. Each mode of the corrugation is characterized by a sinusoid. It is admitted that a sum of k sinusoids fits to an AR model of order $n>k$ (Marple, 1987), some of the poles being allocated to the embedding noise. In most applications the embedding noise is the measurement noise.

The vector a of AR coefficients is computed using a least-square regression. The poles of the AR model are then computed by taking the zeroes of the polynomial with coefficients a.The wavelengths of the corrugation modes are eventually estimated as the inverse of the frequencies of the poles. Only positive frequencies are considered for physical significance.

If the data fit well to the underlying model the AR method is a highly accurate wavelength estimator. However it is commonly admitted that it is not a reliable power spectral estimator. We present in next section the so-called Capon spectral estimator which exhibits better performances.

The Capon filter

The Capon spectral estimator (Capon, 1969) was called a *high-resolution* method at the time it was introduced because it exhibits a sharper spectral profile than the common Fourier-based methods. The Capon method is still of use nowadays because of its high amplitude or power spectral estimation accuracy.

The principle of the Capon method is to design a set of filters with central angular frequencies ω. The filkterbank is such that at each frequency ω,the spectral content of the output of the filter is constrained to be equal to the spectral content of the input. In addition, the spectral content outside the filter with central frequency ω is minimized.

Unlike the AR approach, the Capon method is not a parametric spectral estimation method. It belongs to the family of matched-filterbank methods (Stoica & Moses, 1997). The gain of performance of the Capon method with respect to the Fourier-based methods resides in the additional information provided by the data-dependency of the filterbank design process.

The ARCAP method

Both the AR and the Capon methods require the computation of the inverse of the data covariance matrix. Actually there is a close link between these two methods. Stoica and Moses (Stoica & Moses, 1997) show that the inverse of the Capon power spectral estimator i the sum of the inverses of the AR spectral estimates of order 0 to L, where L is the length (in sample) of the analysed segment. Combining these two approaches is thus quite sensible.

Principle

The ARCAP method proposed by Padovese et al. (1996) combines the highly accurate AR wavelength estimation to the efficient Capon power estimation in order to take advantage of both methods. The algorithm, as proposed by Padovese et al. consists of two steps:

1. <u>AR step:</u> estimate the mode wavelengths from the computed poles of the AR model;
2. <u>Capon step:</u> estimate the spectral power (or magnitude) at each AR wavelength using the Capon method.

In order to get access to the mileage information, the algorithm is performed on contiguous short-length segments of the analysed signal in a fashion similar to what is performed with the spectrogram. Once the wavelength and magnitude of the modes of the segment centred on mileage index *d* have been estimated, the algorithm is applied again on the segment with index *d+1*. The algorithm stops at the final mileage index of the analysed signal.

Recursive implementation

The most computationally demanding operation is the covariance matrix inversion. Note that for a given segment of mileage index *d*, since the covariance matrix has been inverted during the AR step it is not necessary to perform this operation again during the Capon step.
Moreover, Martin has proposed an adaptive implementation of the AR method based on an update equation of the inverse covariance matrix (Martin, 1986). In this approach only the covariance matrix of the first segment is computed and inverted. Then, the inverse matrix of the segment centred on any index *d+1* is computed from the inverse matrix of the previous segment centred on index *d*. The ARCAP method is thus a recursive algorithm that can be efficiently implemented. The update equation reduces the computation load.

Comparison with the spectrogram

Note that the recursive implementation of the ARCAP method is a specificity since the spectrogram and most of the existing time-frequency methods are a collection of stacked spectra which are independently computed. Moreover thanks to this updating step information extracted from the previous signal segment is propagated to the extraction of the information encoded in the current segment.
The spectrogram is computed on a frequency (wavelength) grid that is defined in advance. The wavelength modes are then estimated as the argument of the maxima of the spectrogram. The numerical accuracy of the spectrogram approach is thus bounded by the precision of the chosen frequency grid. In the ARCAP framework, the frequencies are estimated from the computation of the AR coefficients. The accuracy of the ARCAP frequency estimator is thus only bounded by the numerical accuracy of the computer.
Note that there exist several configurations of the Capon filter (Stoica & Moses, 1997). From this method can be derived a power spectral estimator as well as power spectral density estimator such as the periodogram. But the Capon filter can also be used to estimate the amplitude spectrum. This is the configuration we chose because it gives a straightforward access to the depth of the corrugation modes.

Example

Figure 8 shows the ARCAP map of the signal of Figure 4 computed assuming an AR model of order 8. The x-axis is the mileage index. The y-axis is the wavelength of the estimated poles. The colour scale encodes the Capon estimates of the wavelength magnitudes.
The main component in terms of depth is the 100mm component. The estimated depths of the two other components are much smaller than for the corrugation mode. This tends to prove that these two components are related to the measurement noise. They are outliers resulting from the assumed order of the AR model.

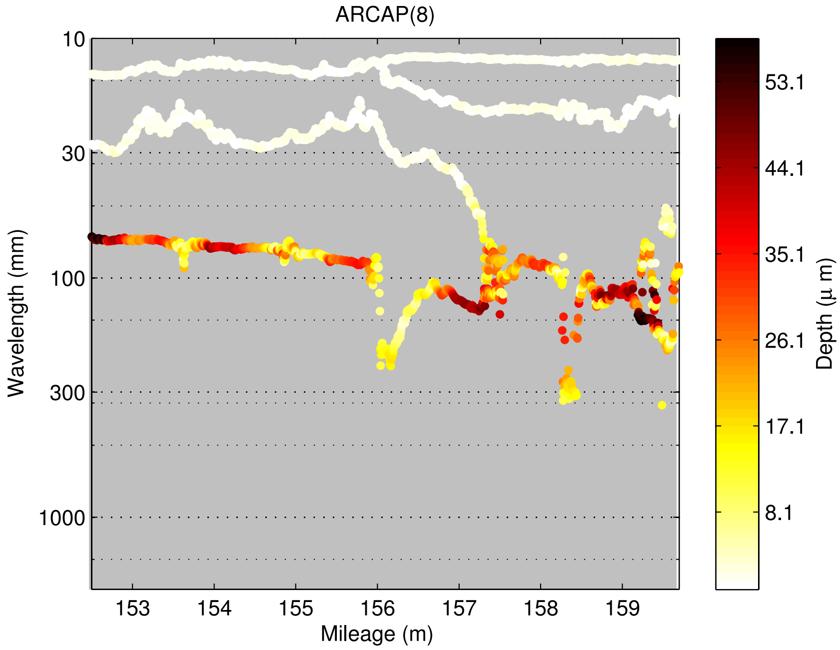

Figure 8: ARCAP map of the signal of Figure 4. Only the positive polar frequencies are displayed. AR model of order 8 computed on a 1 meter-long sliding window with a 0.995 meter-long overlapping, sampling period: 5mm.

Let focus on the corrugation mode. It has been successfully tracked by the ARCAP algorithm when the magnitude is high. When the magnitude collapses (around 156m for instance), the wavelength is dramatically overestimated but the estimated magnitude decreases also. Despite these local outliers, the overall tendency of the corrugation wavelength to increase with mileage already pointed at on the spectrogram of Figure & is well detected by the algorithm.
For diagnosis purpose a threshold can be applied to the ARCAP map. Figure 9 shows the ARCAP after applying a threshold of 0.02mm. The observed outliers are removed and only the corrugation component remains.

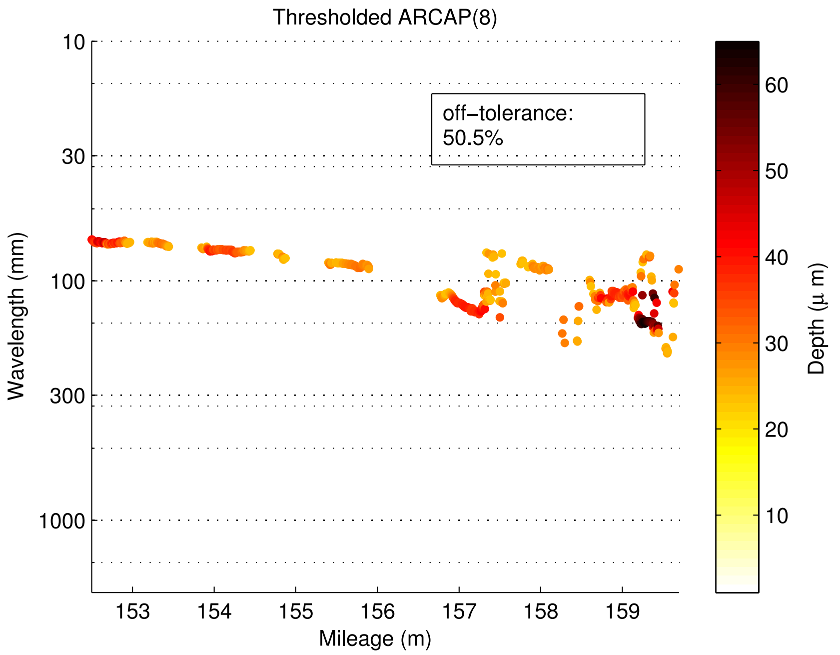

Figure 9: ARCAP map after applying a threshold of 0.02mm on each detected component. 50.5% of the total section length (8.19m) has been detected off-tolerance.

Note that the 600mm component that is observed on the spectrogram of Figure 7 has not been detected by the ARCAP method because this component which is due to the sleepers can not be modelled by an AR process.
This example illustrates the capability of the ARCAP method to detect and track corrugation components. Figure 9 is an example of how corrugation signal can be processed in a fashion similar to the recommendations of the European standards. It is possible to set a tolerance level on the depth of individual corrugation modes and check refurbishment compliance to the standard by applying a threshold to the ARCAP map.
This method allows to evaluate the percentage of section off-tolerance. In addition it allows to accurately localize the wear patterns both along the rail and in terms of wavelength.

CONCLUSION

Rail corrugation is a wear of the rail due to the interaction between bogies and track. Existing methods developed to cure corrugation consist in performing a grinding of the surface of the rail. Even though efficient this solution is highly expensive and time-consuming. Signal processing approaches are thus crucial for performing sensible diagnosis of corrugation triggering the refurbishment procedure.

We have proposed to apply a time-frequency analysis tool to corrugation diagnosis. We proposed to extend the state-of-the-art methodology which consists in a mileage domain or wavelength domain processing of the measurement signal. We have first shown the limitations of these approaches which are enable to encode simultaneously the mileage and wavelength domains information.
We then described the time-frequency methodology chosen based on the application of the ARCAP method. We have justified this choice with modelling considerations.
We have illustrated the possible improvement allowed by the ARCAP method for the diagnosis of corrugation. The ARCAP method provides an easy-to-read mileage-wavelength representation of the analysed signal. Corrugation modes are first detected with high accuracy. Once each corrugation mode is identified the corrugation magnitude of each mode is efficiently estimated.
This approach allows for very accurate localization of the corrugation on rail. It also allows for an accurate characterization of the corrugation modes in terms of wavelength and magnitude.

We have shown that the ARCAP method is an efficient tool for corrugation mode detection and characterization because the assumed underlying data model fits well to corrugation patterns. However other kinds of rail patterns are likely to be present along with corrugation because of the network structure or because of breaks and wear. Such patterns may reduce the performances of the ARCAP method. Moreover, the detection of some structural patterns such as joints can be of interest for other network monitoring tasks such as localisation.

The ARCAP method is by definition a recursive time-frequency approach which can be implemented in a sequential fashion. For industrial implementation matters, it is of interest to develop an on-line version of the algorithm that could be embedded on the measurement device in order to perform on-line corrugation processing.

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